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Fixed Poles in Compton Scattering – Light-Cone Approach*

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The algebra of currents on the light cone is extended to the *covariant* retarded product of these currents. This extension is applied to the study of fixed poles in elastic Compton scattering and in inelastic Compton scattering.

It is possible to establish the asymptotic limit of the imaginary parts of current scattering amplitudes by postulating the singularity structure of a product of such currents on the light cone.[1] In the case of sufficiently convergent amplitudes these results may be turned into sum rules. A complementary approach yielding these results has been the parton idea.[2] In the case of less convergent amplitudes, specifically those requiring subtractions, it is more difficult to obtain any results about the real parts. It is of interest to see whether such models could be extended to real parts, especially to the residues of fixed Regge poles dominating the real parts. Within the context of the parton model some of the results presented in this article have been obtained by Brodsky, Close, and Gunion.[3] Likewise, Cornwall, Corrigan, and Norton[4] obtained answers by making specific assumptions about the nature of subtractions in a Deser-Gilbert-Sudarshan (DGS) representation.

The results alluded to above concern the $J=0$ fixed pole in the $T_1(\nu, q^2)$ amplitude of massive photon Compton scattering, or its analog for the scattering of other SU(3) currents. We shall obtain these results by *postulating* the singularity structure for the covariant retarded product of two currents near the light cone. These assumptions give a prescription for obtaining the covariant time-ordered product from the usual noncovariant one. The reverse procedure has been formulated by Bjorken.[5] This argument may be extended to the more complex situation of inelastic Compton scattering,[6] where the results obtained by Brodsky and Roy[7] in the parton model are obtained from the light-cone approach.

We make the usual ansatz for the commutator of two currents on the light cone[1]:

$$\frac{i}{2\pi}[j_\mu(y), j_\nu(0)] \to -(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\epsilon(y_0)\delta(y^2)B_L(y;0)$$
$$+[g_{\mu\alpha}g_{\nu\beta}\partial^2 - \partial_\alpha(g_{\mu\beta}\partial_\nu + g_{\nu\beta}\partial_\mu) + g_{\mu\nu}\partial_\alpha\partial_\beta]\epsilon(y_0)\theta(y^2)B_2^{\alpha\beta}(y;0)\,, \tag{1}$$

where $B_L(y;0)$ and $B_2^{\alpha\beta}(y;0)$ are bilocal operators regular at $y=0$. The spin-averaged forward proton matrix elements of these operators are

$$(2\pi)^3\frac{E}{M}\langle p|B_L(y;0)|p\rangle = f_L(y\cdot P, y^2)\,,$$
$$(2\pi)^3\frac{E}{M}\langle p|B_2^{\alpha\beta}(y;0)|p\rangle = p^\alpha p^\beta f_2(y\cdot P, y^2) + \cdots\,, \tag{2}$$

where the omitted terms do not contribute to the order we are interested in. The usual structure functions of inelastic electron scattering,

$$W_{\mu\nu}(\nu, q^2) = (2\pi)^2\frac{E}{M}\int e^{iq\cdot y}\langle p|[j_\mu(y), j_\nu(0)]|p\rangle\, d^4y$$
$$=\frac{1}{M^2}\left(p_\mu - \frac{M\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{M\nu}{q^2}q_\nu\right)W_2(\nu, q^2) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1(\nu, q^2)\,, \tag{3}$$

with $M\nu = p\cdot q$, may be obtained in the limit $\nu, q^2 \to \infty$ with $x=-q^2/2M\nu$ fixed from the functions f_L, f_2:

$$\nu W_2 \to 4\pi i x\int d(y\cdot p)y\cdot p\, e^{-ixy\cdot p}f_2(y\cdot p, 0)\,, \tag{4a}$$

$$W_1 = 2\pi \int d(y\cdot p)\, e^{-ixy\cdot p}[xf_L(y\cdot p, 0) + iy\cdot p f_2(y\cdot p, 0)]\,. \tag{4b}$$

We are, however, interested in the full amplitude

$$T_{\mu\nu} = -i(2\pi)^3 \frac{4E}{M} \int e^{iq\cdot y}\theta^*(y_0)\langle p|[j_\mu(y), j_\nu(0)]|p\rangle\, d^4y\,, \tag{5}$$

where $\theta^*[\ ]$ denotes the covariant retarded commutator. The ordinary retarded commutator $\theta[\ ]$ may be obtained from Eq. (1) just by placing the step function $\theta(y_0)$ in front of the right-hand side of this equation. However, $\theta(y_0)\partial\cdots\partial\epsilon(y_0)\delta(y^2)$ or $\theta(y_0)\partial\cdots\partial\epsilon(y_0)\theta(y^2)$ are not Lorentz-invariant scalars, and thus the covariant and ordinary retarded products will differ. We shall make the simplest *assumption*[8] on the structure of the covariant product: Place $\theta(y_0)$ to the right of all spatial derivatives and drop any polynomial in $\partial^{\mu_1}\cdots\partial^{\mu_n}\delta^4(y)$:

$$\frac{i}{\pi}\theta^*(y_0)[j_\mu(y), j_\nu(0)] \to -(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\theta(y_0)\delta(y^2)\, B_L(y;0)$$
$$+[g_{\mu\alpha}g_{\nu\beta}\partial^2 - \partial_\alpha(g_{\mu\beta}\partial_\nu + g_{\nu\beta}\partial_\mu) + g_{\mu\nu}\partial_\alpha\partial_\beta]\theta(y_0)\theta(y^2)B_2^{\alpha\beta}(y;0)\,. \tag{6}$$

We have no justification for this assumption except that it is simple and yields the same results as the cutoff field-theory calculations of Ref. 3.

The limit we shall study is the one in which ν becomes large with q^2 fixed.[9] Unfortunately, even the assumptions of Eq. (6) are not sufficient. In the indicated limit $T_{\mu\nu}$ becomes

$$T_{\mu\nu} \to +\frac{4(2\pi)^4 E}{M\nu}\int e^{i\nu(t-r)}e^{-iMxr}\theta^*(t)\langle p|[j_\mu(t,\vec{r}), j_\nu(0)]|p\rangle\, r\,dr\,dt\,. \tag{7}$$

The large-ν, fixed-q^2 limit receives contributions from two regions of the t, r integration. We assume that the first, $|r-t|<1/\nu$ and r large, contributes to the ordinary Regge limit, while the second, consisting of the neighborhood of the light cone, contributes to fixed Regge poles at integer J. We shall be interested in this second term. In order to discuss this contribution, especially the one building a fixed pole at $J=0$, it is necessary to subtract all terms of type one which dominate the term of interest, specifically all Regge singularities at $J>0$.[10] This procedure has been discussed extensively in Refs. 3, 4, and 10, and for simplicity we shall ignore this complication by assuming that the pole at $J=0$ makes the dominant contribution to $T_{\mu\nu}$. Combining Eqs. (5), (6), and (7) we find

$$T_{\mu\nu} \to 4i\frac{(2\pi)^2}{\nu}(q_\mu q_\nu - g_{\mu\nu}q^2)\int \theta(t)\delta(t^2-r^2)\, e^{i\nu(t-r)}f_L(Mt,0)r\,dr\,dt$$
$$+4i\frac{(2\pi)^2}{\nu}[p_\mu p_\nu - M\nu(p_\mu q_\nu + p_\nu q_\mu) + M^2\nu^2 g_{\mu\nu}]\int \theta(t)\theta(t^2-r^2)\, e^{i\nu(t-r)}f_2(Mt,0)r\,dr\,dt\,. \tag{8}$$

In the indicated limit we note that T_1 (the coefficient of $-g_{\mu\nu}$) approaches

$$T_1 \to 4(2\pi)^2\int_0^\infty p\cdot y\, d(p\cdot y) f_2(p\cdot y, 0)\,. \tag{9}$$

Contingent on the conditions discussed above regarding the $J=0$ pole dominating the amplitude (this is equivalent to having $\nu W_2/x^2$ finite at $x=0$) we obtain from Eq. (9)

$$T_1 \to -2\int \frac{\nu W_2}{x^2}\, dx\,, \tag{10}$$

which is the result of Ref. 3. (The support of the x integration may be obtained from spectral conditions.)

We now turn to inelastic Compton scattering, $\gamma + p \to \gamma + \text{anything}$, and note what contributions this fixed pole makes to this amplitude. Let k_i (k_f) and ϵ_i (ϵ_f) be the momentum and polarization of the initial (final) real photon. The cross section for detecting the outgoing photon is

$$d\sigma = \frac{d^3k_f}{2\omega_f\omega_i}\frac{M}{E_p}\frac{1}{(2\pi)^3}A\,, \tag{11}$$

where

$$A=-(2\pi)^3\frac{E}{M}\int d^4z_0d^4z'd^4w\, e^{ik_+\cdot(z-z')/2}e^{-ik_-\cdot w}\epsilon_i^\mu\epsilon_i^\beta\epsilon_f^\nu\epsilon_f^\alpha$$
$$\times\langle p|\,\theta^*(z')[j_\alpha(\tfrac{1}{2}z'),j_\beta(-\tfrac{1}{2}z')]\theta^*(z)[j_\nu(\tfrac{1}{2}z+w),j_\mu(-\tfrac{1}{2}z+w)]\,|p\rangle\ , \tag{12}$$

with $k_\pm=k_i\pm k_f$, $k_-\cdot p=M\nu$, $k_+\cdot p=M\mu$, $k_-^{\,2}=t$.

We investigate the limit μ, ν, $t\to\infty$, $\nu/\mu\to 0$; $t/\mu\to 0$ and $-t/2M\nu=x$ fixed. This is the limit studied in Ref. 7. Again we must make the critical assumption that the fixed pole dominates the retarded product. In this situation this is plausible, as t is large and thus all other moving singularities may have moved to J values less than zero. If any singularities with $J>0$ do survive this large-t limit, as for instance a Pomeranchukon trajectory, then the following discussion will be invalid as will be the arguments in 6 and 7.

With the above warnings in mind the term A of Eq. (12) tends to

$$A\to+\frac{(2\pi)^3E}{4M}(\epsilon_i\cdot\epsilon_f)^2k_{+\alpha}k_{+\beta}k_{+\mu}k_{+\nu}\int d^4z\,d^4z'\,d^4w\,e^{ik_+\cdot[(z-z')/2]}e^{-ik_-\cdot w}$$
$$\times\theta(z_0')\delta(z'^2)\,\theta(z_0)\delta(z^2)\langle p|\,B_2^{\alpha\beta}(\tfrac{1}{2}z',-\tfrac{1}{2}z')B_2^{\mu\nu}(\tfrac{1}{2}z+w,-\tfrac{1}{2}z+w)\,|p\rangle\ . \tag{13}$$

To proceed further we would have to assume an algebra of bilocal operators on the light cone. The only one considered is the one based on the light cone,[11] which reproduces in detail the parton calculation of Ref. 7. In Ref. 7 it is argued that the conditions $\nu/\mu\to 0$ and $t/\mu\to 0$ are essential for the validity of the parton calculations. This point was not emphasized in Ref. 6. In the present development this limit is essential to ensure the dominance by the terms on the light cone that build up the fixed pole.

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